C1 Jan 2011

1.

(a) Find the value of $16^{-\frac{1}{4}}$ **(2)**

(b) Simplify $x\left(2x^{-\frac{1}{4}}\right)^4$ **(2)**

2.
Find

$$\int \left(12x^5 - 3x^2 + 4x^{\frac{1}{3}}\right) \mathrm{d}x$$

giving each term in its simplest form. **(5)**

3.
Simplify

$$\frac{5-2\sqrt{3}}{\sqrt{3}-1}$$

giving your answer in the form $p + q\sqrt{3}$, where p and q are rational numbers. **(4)**

4.
A sequence $a_1,\ a_2,\ a_3, \ldots$ is defined by,

$$a_1 = 2$$
$$a_{n+1} = 3a_n - \mathrm{c}$$

where c is a constant

(a) Find an expression for a_2 in terms of c. **(1)**

Given that $\sum_{r=1}^{3} a_r = 0$

(b) find the value of c **(4)**

5.

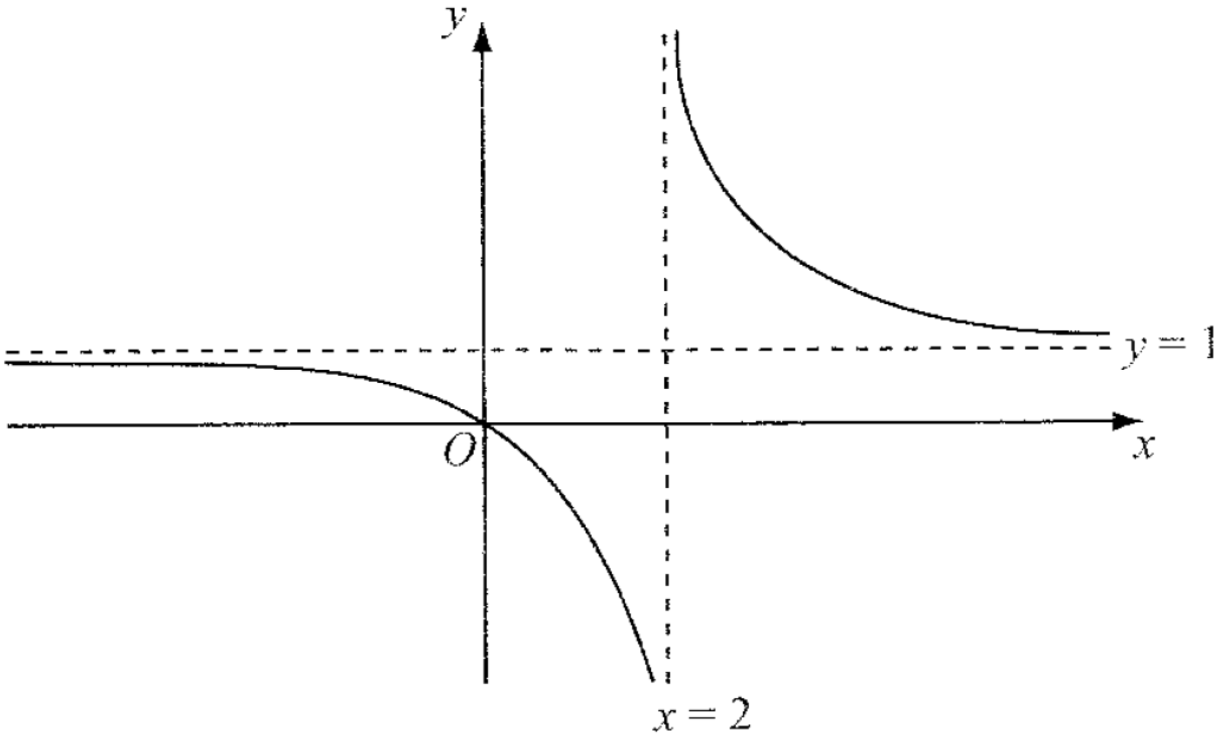

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{x-2}, \qquad x \neq 2$$

The curve passes through the origin and has two asymptotes, with equation $y = 1$ and $x = 2$, as shown in figure 1.

(a) In the space below, sketch the curve with equation $y = \mathrm{f}(x-1)$ and state the equations of the asymptotes of this curve. **(3)**

(b) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x-1)$ crosses the coordinate axes **(4)**

6.
An arithmetic sequence has first term a and common difference d. The sum of the first 10 terms of the sequence is 162.

(a) Show that $10a + 45d = 162$ **(2)**

Given also that the sixth term of the sequence is 17,

(b) write down a second equation in a and d, **(1)**

(c) find the value of a and the value of d **(4)**

7.
The curve with equation $y = \mathrm{f}(x)$ passes through the point (-1, 0).

Given that

$$\mathrm{f}'(x) = 12x^2 - 8x + 1$$

find $\mathrm{f}(x)$. **(5)**

8.
The equation $x^2 + (k-3)x + (3-2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies **(3)**

$$k^2 + 2k - 3 > 0$$

(b) Find the set of possible values of k. **(4)**

9.
The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point $A(1, 4)$lies on L_1 , find

(a) the value of k **(1)**

(b) the gradient of L_1 **(2)**

The line L_2 passes through A and is perpendicular to L_1 .

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers **(4)**

The line L_2 crosses the x-axis at point B.

(d) find the coordinates of B **(2)**

(e) find the exact length of AB **(2)**

10.

(a) On the same axes, sketch the graphs of

$(i)\quad y = x(x+2)(3-x)$

$(ii)\quad y = -\frac{2}{x}$

showing clearly the coordinates of all the points where the curve cross the coordinate axes.

(6)

(b)Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x+2)(3-x) + \frac{2}{x} = 0$$

(2)

11.

The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

(4)

(b)Show that the point $P(4, -8)$lies on C

(6)

(c)Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers

(2)